A study of a scalar field probes micro space-time

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Abstract

In this work, we try to find a way to describe the physical law of micro-world under the frame of a space-time theory. By introducing a scalar field $\mathcal{D}(x)$, we rewrite the action of conventional field theory and the Lagrangian describing the motion of the particle, where a modified space-time relation is obtained. To prove the correctness of this attempt, we derive the Klein-Gordon equation by the Hamilton-Jacobi method in four dimensional form.

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I. INTRODUCTION

General relativity (GR) [1, 2] is a milestone in 20th century physics. It reveals the nature gravity and greatly promotes the development of modern cosmology. In particular, the discovery of the accelerated expansion of the universe led to the emergence of various modified theories of gravity [3–10], these theories have made important progress in solving the problem of cosmological constant, inflation, or structure formation, etc. As one of the famous modified theories, the scalar-tensor theory has attracted widespread attention. As we know, conventional general relativity is a geometrical theory or a metric theory, which is also called " tensor theory ". Although usually we should set the degrees of freedom in the theory as few as possible, it does not rule out the introduction of additional scalar fields. In the original scalar tensor theory, the scalar field was related to a changing gravitational constant, thus the gravity can be adjusted. The sources of scalar fields are various, it could be the dilaton from string theory, the scalar field in a brane world, or comes the size of compactified internal space, etc [11]. This feature of scalar field theory is very attractive because it provides a lot of freedom for the birth of new theories. In the development of modified gravity theories, the variational principle played an important role. Starting from the action amount, the equation of motion of the field or the particle can be easily obtained, the modification of a theory can be obtained by modifying its action.

Since the birth of general relativity, space-time has entered people's field of vision as a special research object, the space-time geometry is closely related to the distribution of matter. Generally, GR is used in large-scale research, especially in cosmology, but when it comes to micro-scale, a modified field equation might be needed to describe the law of physics. In cosmology, especially the fine-tuning problem [12] indicate a gravity theory that can work at a micro scale is urgently needed.

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In this article, we try to use the methods of scalar-tensor theory and the concept of space-time to derive the law of physics in the microcosm. Generally, such an attempt is difficult. however, the introduction of the concept of space-time in GR, especially the view that space-time has a microstructure in some quantum gravity theories [13–21], provides a feasible way for this attempt. For technical details, we take a view that some microstructures of space-time would induce a scalar field $\mathcal{D}(x)$. Considering a particle moving in the background with such a scalar field, we propose the action of fields and the Lagrangian describing the motion of the particle. According to the action, one can get the relationship between the field $\mathcal{D}(x)$ and another scalar field $\zeta(x)$. With the Hamiltonian discussion, a modified space-time relation in general relativity is obtained. In order to test the practicability in describing the physical law in micro space-time, we demonstrate the Klein-Gordon equation [22–24] by the basic equations from this work. In this part, the four dimensional Hamilton-Jacobi method is chosen. Throughout this paper we use the signature $(-,+,+,+)$.

II.   THE BASIC EQUATIONS

Considering a spin-independent particle moving in an arbitrary background, we choose the action of fields as

$$I = I_0 + \int \sqrt{-g}\lambda \left[-\frac{1}{2}g^{\mu\nu}\partial_\mu\zeta\partial_\nu\zeta + \frac{1}{2}\left(\frac{mc}{\hbar}\right)^2 \mathcal{D}\zeta^2 \right] d^4x, \tag{1}$$

and the Lagrangian describing the motion of the particle is

$$L = L_0 - \frac{1}{2}mc^2\mathcal{D}. \tag{2}$$

Where I_0 can be the action of arbitrary theories we usually known, such as gauge theory [25–30] or various gravity theories. L_0 is the Lagrangian of the particle in arbitrary known theories and m is the proper mass of the particle. c is the speed of

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light and $\hbar$ is the reduced Planck constant (For convenience, we use natural units in which $c = \hbar = 1$ in the rest of this paper), λ is a constant with the dimension of length. The scalar function $\mathcal{D}(x)$ that we especially propose in this work form some microstructure of space-time represents a proper time field. Finally, $\zeta(x)$ is a scalar field with its direct explanation as

$$\zeta^2 = \eta, \tag{3}$$

here η is the proper space density of the particle caused by restricting the space of its existence, it can be also understood as the proper probability density because of the proportional relationship between two explanations. Thus it is reasonable to require the space density to satisfy the conservation law

$$\partial_\mu \left(\sqrt{-g} \eta u^\mu \right) = 0. \tag{4}$$

Similar to what we have done in conventional theories, we require that the space density of the existence be positive continuous, finite and single-valued at every point of space.

In order to further understand above formulas, we can write Eq. (1) and Eq. (2) more specifically. For example, considering a spin-independent particle moving in the electromagnetic field under Einstein gravity, with all elements the action and the Lagrangian take the form

$$I = \int \sqrt{-g} \left[\frac{1}{16\pi G} R - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \mathcal{L}_m + \lambda \left(-\frac{1}{2} g^{\mu\nu} \partial_\mu \zeta \partial_\nu \zeta + \frac{1}{2} m^2 \mathcal{D} \zeta^2 \right) \right] d^4 x, \tag{5}$$

$$L\left(x^\mu, \dot{x}^\mu\right) = \frac{1}{2} m g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu + q A_\mu \dot{x}^\mu - \frac{1}{2} m \mathcal{D}. \tag{6}$$

Where R is the curvature scalar, the electromagnetic tensor $F_{\mu\nu}$ satisfies $F_{\mu\nu} = A_{\nu;\mu} - A_{\mu;\nu}$ and A_μ is the four-component vector potential. $\mathcal{L}_m$ is the matter Lagrangian

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density which provides the source of gravitational field and electromagnetic field. In Eq. (6), $\dot{x}^\mu = \frac{dx^\mu}{d\tau} = u^\mu$ is the four-velocity, τ is the proper time of the particle and q is the charge of the particle. For such a particle, the Hamiltonian is

$$H = \pi_\mu \dot{x}^\mu - L = \frac{1}{2}m\left(g_{\mu\nu}u^\mu u^\nu + \mathcal{D}\right), \tag{7}$$

where

$$\pi_\mu = \frac{\partial L}{\partial \dot{x}^\mu} = mu_\mu + qA_\mu \tag{8}$$

is the four canonical momentum vector. Then the conservation of the Hamilton Eq. (7) leads to

$$g_{\mu\nu}u^\mu u^\nu + \mathcal{D} = const. \tag{9}$$

When $\mathcal{D} = 0$, Eq. (9) should go back to the results of conventional gravity theory, that is

$$g_{\mu\nu}u^\mu u^\nu = -1, \tag{10}$$

thus we can determine the constant and re-express Eq. (9) as

$$g_{\mu\nu}u^\mu u^\nu + \mathcal{D} = -1. \tag{11}$$

To make a clear explanation to the above formula, let's rewrite it as:

$$g_{\mu\nu}dx^\mu dx^\nu = -d\bar{\tau}^2, \tag{12}$$

and

$$d\bar{\tau} = \sqrt{1 + \mathcal{D}}d\tau. \tag{13}$$

Here $\bar{\tau}$ is the proper time of the local coordinate which moves together with the particle. Now we can see that the space-time relation of the coordinate in this work remains same to the convention of GR, but the proper time of the particle is not equal to the proper time of the coordinate because there is a proper time field, therefore,

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the comprehension of the four velocity and the proper space density of the existence of the particle also need to be justified by this way.

To get more understanding of the proper time field $\mathcal{D}(x)$, we can also pay attention to the equation of the scalar field ζ. Varying the action Eq. (5) with respect to ζ, we get

$$\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\zeta\right) + m^2\sqrt{-g}\mathcal{D}\zeta = 0, \tag{14}$$

this equation shows that the relation between proper time field and the space density field, and it constitutes the basic equations of this work together with Eq. (4) and Eq. (11).

III.   KLEIN-GORDON EQUATION

In order to test the correctness of the basic equations in describing physical law at the microscopic scale, the Hamilton-Jacobi method [31] is chosen:

$$H\left(x^\mu, \partial_\mu S\right) + \frac{\partial S}{\partial \tau} = 0, \tag{15}$$

where S is the Hamilton-Jacobi function and π_μ has been substituted by $\partial_\mu S$. With

$$S = W\left(x^\mu, \pi_\mu\right) - a_h\tau, \tag{16}$$

one has

$$H\left(x^\mu, \partial_\mu W\right) = a_h = -\frac{1}{2}m. \tag{17}$$

The quantity W is Hamilton characteristic function and $\pi_\mu = \partial_\mu W$, now Eq. (11) can be rewritten as

$$g^{\mu\nu}\left(\partial_\mu W - qA_\mu\right)\left(\partial_\nu W - qA_\nu\right) + m^2\mathcal{D} = -m^2, \tag{18}$$

and Eq. (4) can also get a new form in the same way:

$$\partial_\mu\left[\sqrt{-g}\zeta^2 g^{\mu\nu}\left(\partial_\nu W - qA_\nu\right)\right] = 0. \tag{19}$$

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The equations Eq. (18), Eq. (19) and Eq. (14) can be the new basic formulas in describing the motion of a particle moving in the electromagnetic field under gravity.

To reach the task, we can introduce a new function ξ by defining:

$$W = \frac{\hbar}{i} \ln \xi, \tag{20}$$

substituting it into Eq. (18) and Eq. (19), then combining with Eq. (14), one can get

$$\frac{1}{\sqrt{-g}} \partial_\mu \left[\sqrt{-g} g^{\mu\nu} \left(\partial_\nu - iqA_\nu \right) \phi \right] - g^{\mu\nu} iqA_\mu \left(\partial_\nu - iqA_\nu \right) \phi - m^2 \phi = 0, \tag{21}$$

this is the Klein-Gordon equation derived from this theory. Here $\phi = \xi\zeta$ is the wave-function of the particle.

As for the positive definiteness of the probability density, let us define $\gamma = \frac{dt}{d\tau}$, regarding to Eq. (4) and $\phi^*\phi = \eta$, then the equation of normalization can be written as

$$\int_\infty \sqrt{-g} \gamma \phi^* \phi d^3 x = 1. \tag{22}$$

Now we can see that the wave-function in the Klein-Gordon equation has clearly got its explanation.

According to the above formulas, the practicability of the basic equations from this work in describing the physical law of microscopic space-time is proved. On the other hand, Eq. (21) may be also used as a convenient way to calculate $\mathcal{D}(x)$, which plays a major role in this theory.

IV. CONCLUSIONS AND OUTLOOK

In this paper, in order to probe the physical law in micro-world under the frame of space-time theory, we introduce a scalar field $\mathcal{D}(x)$, then a modified space-time relation is obtained. One can see that the proper time field $\mathcal{D}(x)$ relates to the

probability density field. Also, using the Hamiltonian-Jacobi equation, the Klein-Gordon equation was obtained by the basic equations in this work, which shows this probe of micro space-time is effective.

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